$$\frac{d\,\mathrm{Re}}{d\bar{t}} = \frac{\mathrm{Re}}{2\bar{r}^2}(\mathrm{Re} - 8) + \alpha^2, \quad \frac{d\bar{r}}{d\bar{t}} = -\frac{\mathrm{Re}}{\bar{r}} \tag{6}$$

with the initial conditions Re (0) = 0 and $\bar{r}(0) = 1$. In the case of an arbitrary dependence $p_\infty(t, \tau)$ one determines Re_τ numerically.

Let us consider the limiting case when the pressure pulse retains finite as $\tau \to 0$.

The solution of the system (6) has the form

$$\mathrm{Re}_\tau = I/\rho\nu, \ \bar{r} = 1.$$

For complete collapse of the cavity it is necessary that $I \geq \mathrm{Re}_* \rho\nu$.

Partial collapse of the cavity occurs when $I < \mathrm{Re}_* \rho\nu$, where

$$r_{\lim}/r_0 = (1 - I/\mathrm{Re}_*\rho\nu)^2.$$

For rectangular pressure pulses

$$p_\infty(t, \tau) = p_0 U_-(\tau - t), \ p_0 = \mathrm{const}$$

the quantity Re_τ essentially depends on the parameter $\alpha = \sqrt{p_0/\rho}(r_0/\nu)$: it either grows without limit with an increase in τ or, reaching a maximum at some τ, subsequently approaches zero. The motion of a cavity in a viscous liquid under the action of a constant pressure was studied in [1] and a critical value of $\alpha_* = 8.4$ was obtained for the parameter. When $\alpha > 8.4$ the velocity of the cavity boundary grows without limit as $r^{-3/2}$ with a decrease in radius, and therefore the Reynolds number also grows without limit: $\mathrm{Re} = |u_1| r_1/\nu \sim r_1^{-1/2}$. From the law (2) of variation in the velocity in the absence of external pressure it follows that when $\alpha < 8.4$ the maximum value is $\mathrm{Re}_\tau < \mathrm{Re}_*$ and partial collapse of the cavity occurs at any finite values of the pressure pulse. The dependence of Re_τ on the pressure pulse $I = p_0\tau$, obtained through numerical integration of the system (6), is presented in Fig. 2.

For $\alpha > 8.4$ there is a minimum value of the pressure pulse $I_{\min}$ at which the cavity collapses, in which case $\mathrm{Re}_\tau = \mathrm{Re}_*$. The dependence of $I_{\min}$ on α is presented in Fig. 3.

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DIMENSIONLESS EQUATIONS OF STATE AND ATTENUATION OF SHOCK WAVES

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1. Dimensionless Hugoniot Equations of State

Basic information as regards the compressibility of materials and their thermodynamics at high pressures is at present obtained from shock-wave experiments [1]. By using the wave velocity D and the mass velocity U, the pressures (as well as densities and specific energies) are made constant in them for determining the path of the Hugoniot adiabat. The remarkable empirical relation found in a number of experiments consists in that for many materials a linear dependence is observed between the shock-wave velocity and the downstream velocity of the matter, $D = C_0 + \lambda U$. This relation, together with the conservation laws, yields straightforward expressions for the shock pressure P_H, for the increase of the inner energy $E_H - E_0$, and for the deformation X:

$$X = 1 - \rho_0/\rho = U/(C_0 + \lambda U), \quad P_H = \rho_0 C_0^2 X/(1 - \lambda X)^2,$$
$$E_H - E_0 = 0.5 C_0^2 X^2/(1 - \lambda X)^2, \tag{1.1}$$

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where ρ is the density; the subscript 0 refers to the upstream front states of the wave and H, to the downstream ones.

The state equation of the Mie–Grüneisen type associated with the Hugoniot adiabat can, in view of (1.1), be written as [2-4]

$$P(X,E) = P_H(X) + \frac{\rho_0 \Gamma(X)}{1-X}[E - E_H(X)], \tag{1.2}$$

where $\Gamma(X)$ is the Grüneisen coefficient. Moreover, in agreement with the experiments [5], one sets $\Gamma\rho = \Gamma_0\rho_0 =$ const. In the thermal variables X, T, if the heat capacity is constant, one has

$$P(X,T) = P_H(X) + \frac{\rho_0 c_V \Gamma(X)}{1-X}[T - T_H(X)]. \tag{1.3}$$

The temperature on the shock adiabat T_H is determined by the temperature T_0 of the initial state and by the solution of a differential equation [4, 5].

If the energy is measured from the initial state, that is, if one assumes that $E_0 = 0$, then Eqs. (1.2) and (1.3) together with (1.1) contain only five parameters – ρ_0, C_0, λ, c_V, T_0 – and a single density function $\Gamma(X)$. The latter enables one to describe the thermodynamics of the compressed matter by means of universal relations. The first step in this direction was made in [6, 7] by introducing the pressure scale $\rho_0 C_0^2$ and the velocity scale C_0, by selecting the values of the coefficient λ and of the Grüneisen coefficient. Much more substantial results were obtained in [8-10], where two different scales were proposed for the velocity: C_0 and C_0/λ. The dimensionless variables introduced in [8] are

$$d = D/C_0,\quad u = \lambda U/C_0,\quad p = \lambda P/(\rho_0 C_0^2),\quad e = \lambda^2 E/C_0^2,\quad x = \lambda X.$$

Then the relations on the shock adiabat assume a very simple form:

$$d = 1 + u,\quad x = u/(1+u),\quad p_H = x/(1-x)^2,\quad e_H = 0.5x^2/(1-x)^2, \tag{1.4}$$

and do not contain any specific parameters. It was shown in [10] that one can introduce a dimensionless temperature and write Eqs. (1.2) and (1.3) in a dimensionless form; we write the relations obtained in [10] in the form

$$p(x,\ e) = x/(1-x)^2 + \gamma_0[e - 0.5x^2/(1-x)^2]; \tag{1.5}$$

$$p(x,t) = x/(1-x)^2 + \gamma_0[t - \theta(x) - t_0 e^{\gamma_0 x}]. \tag{1.6}$$

In the above $t = \lambda^2 c_V^2 T/C_0^2$ is dimensionless temperature; $\gamma_0 = \Gamma_0/\lambda$; $t_0 = \lambda^2 c_V T_0/C_0^2$ is the dimensionless initial temperature at the point (p = 0, x = 0); $\theta(x) = t_H(x) - t_0 e^{\gamma_0 x}$, where t_H is the shock compression temperature; $t_0 e^{\gamma_0 x}$ is the temperature at the isentrope passing through the point (p = 0, x = 0). The function $\theta(x)$ is the solution of the differential equation

$$d\theta(x)/dx = \gamma_0\theta(x) + x^2/(1-x)^3$$

with the initial value $\theta(0) = 0$. For $\theta(x)$ one has the following approximate formula:

$$\theta(x) = \frac{x^3}{3(1-x)^2}[1 + (\gamma_0/4 + 0.293)x] \tag{1.7}$$

in the region $0.8 \le \gamma_0 \le 2$, $0 \le x \le 0.64$, in which the relative error is less than 0.5%.

Thus, all the thermodynamic processes and quantities can be evaluated using the formulas (1.5)-(1.7); in particular, the pressure on the null isentrope $s = s_0$ is

$$p_{s_0}(x) = x/(1-x)^2 - \gamma_0\theta(x).$$

On any isentrope passing through the point $(p_i,\ x_i)$ one has

$$p_{s_i}(x) = p_{s_0}(x) + [p_i - p_{s_0}(x_i)]\, e^{\gamma_0(x - x_i)}. \tag{1.8}$$

For the adiabat of the repeated impact loading which starts at the point $(p_i,\ x_i)$, one has the following relations:

$$u = u_i + (\sigma - \sigma_i)\sqrt{\varphi(\sigma,\ \sigma_i)/(\sigma\sigma_i)},\quad p = p_i + (\sigma - \sigma_i)\varphi(\sigma,\ \sigma_i), \tag{1.9}$$

where $\sigma = 1/(1-x)$; $p_i = p_H(x_i)$;

$$\varphi(\sigma,\sigma_i) = \frac{(\sigma + \sigma_i - 1)[1 - \gamma_0(\sigma - 1)/\sigma] + \gamma_0(\sigma_i - 1)/(2\sigma)}{1 - \gamma_0(\sigma - \sigma_i)/(2\sigma\sigma_i)}.$$

TABLE 1

x	p_H	e_H	$\gamma_0=0,8$ θ	$\gamma_0=0,8$ c	$\gamma_0=1,2$ θ	$\gamma_0=1,2$ c	$\gamma_0=1,6$ θ	$\gamma_0=1,6$ c	$\gamma_0=2$ θ	$\gamma_0=2$ c
0	0	0	0	1,000	0	1,000	0	1,000	0	1,000
0,10	0,123	0,006	0,0004	1,102	0,0004	1,100	0,0004	1,098	0,0004	1,095
0,20	0,312	0,031	0,0046	1,208	0,0046	1,200	0,0047	1,192	0,0048	1,183
0,30	0,612	0,092	0,0210	1,325	0,0216	1,305	0,0221	1,285	0,0228	1,265
0,40	1,111	0,222	0,0708	1,456	0,0732	1,419	0,0757	1,381	0,0784	1,342
0,44	1,403	0,309	0,1100	1,515	0,1140	1,469	0,1182	1,421	0,1227	1,371
0,48	1,775	0,426	0,1685	1,579	0,1749	1,521	0,1818	1,462	0,1891	1,400
0,52	2,257	0,587	0,2558	1,648	0,2659	1,578	0,2768	1,505	0,2885	1,428
0,56	2,893	0,810	0,3866	1,725	0,4025	1,640	0,4195	1,551	0,4378	1,456
0,60	3,750	1,125	0,5851	1,811	0,6095	1,709	0,6359	1,600	0,6644	1,483
0,64	4,938	1,580	0,8912	1,903	0,9287	1,786	0,9693	1,654	1,0133	1,510

The latter enable us to evaluate the second adiabat both in the plane p−x and in the plane p−u.

Finally, the dimensionless isentropic sound velocity c(x, p) is determined by

$$c^2(x, p) = (1-x)^2\left(\frac{\partial p}{\partial x}\right)_s = \gamma_0(1-x)^2 p + (1+x-\gamma_0 x)/(1-x), \qquad (1.10)$$

which is related to the standard isentropic sound velocity C by the relation $c = (1-x)\rho C/\rho_0 C_0$ and is understood to be the propagation velocity of small perturbations (or weak disruptions) in the case of one-dimensional planar motion and also in the case when the corresponding dimensionless space and time coordinates introduced below are employed.

In [10], thermodynamical analysis was carried out for one mean value $\gamma_0 = 4/3$. Our problem consists in a full thermodynamic description of "linear" media with different γ_0 when they are shock compressed or expanded.

2. Shock Adiabats

Dimensionless parameters of shock adiabats calculated by using (1.4), (1.7), and (1.10) are shown in Table 1 for γ_0 = 0.8, 1.2, 1.6, 2. Here p(x) and e(x) for all γ_0 and also the sound velocities and the function θ(x) for finding $t_H(x)$ are given. With γ_0 increasing, the sound velocities are reduced for the same degree of compression, while θ(x) and the temperature on the shock adiabat increase. In agreement with the theory, the quantity θ(x) increases as the third order of smallness with respect to x and it begins to exert an effect on the parameters of shock compression for x > 0.3.

The condition [11] $P = 2\rho_0 C_0^2/\lambda$ or p = 2 is the melting criterion for a shock wave. It can be seen from Table 1 that melting sets in when x > 0.5. It is known [12] that compressibility of a medium at high pressures hardly depends on transition to the liquid phase though the temperature growth is reduced. Of course, the values of θ(x) for x > 0.5 differ from the true values in the direction of the greater ones.

3. Disintegration of Burst and Error of Mirror Approximation

Disintegration of the burst process is considered when the shock wave is reflected from the interface between two media. Such a situation arises in dynamic compressibility by the reflection method [1]. Depending on the relative dynamic rigidity of the layers through which the shock wave is passing, either extension or contraction of the first layer takes place. On the p−u diagram of Fig. 1 the compression branch L and the extension R start at the state (p_H, u_H). These curves of the p−u diagram are, as a rule, identical with the mirror image of the basic adiabat of single compression (dashed line), although they apply to different thermodynamic processes. To find the discharge curves the relation for a simple wave

$$du/dx = c(x, p)/(1-x)$$

is used; this relation can be integrated numerically by using (1.8) and (1.10). The second adiabats in the p−u coordinates are found from Eqs. (1.9).

The results of the calculations for three values of γ_0 − 0.8, 1.2, and 1.6 − are shown on the nomograms in Figs. 2-4. The dimensionless mass velocities u_H are shown on the abscissa axis and on the ordinate axis,

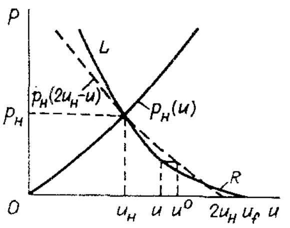

Fig. 1

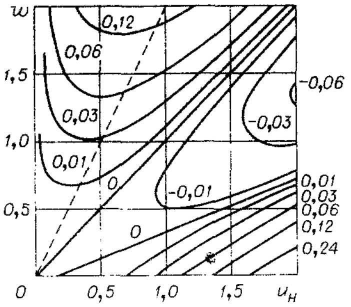

Fig. 2

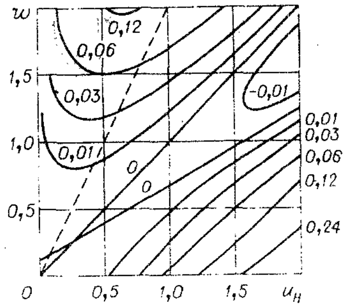

Fig. 3

the quantity $w = 2u_H - u^0$, where u^0 is the velocity after the burst break up has taken place in the mirror approximation. The dashed straight lines correspond to the ordinate in Fig. 1, that is, to the reflection states from an absolutely rigid wall. The result of the analysis is shown by the isolines $u - u^0 = \text{const}$. They characterize the error of the mirror approximation. The values on the curve show by how much the time dimensionless velocity u differs from the values in the mirror approximation following the unloading or secondary compression. The graphs show the applicability domains of the mirror approximation and enable one to introduce the required corrections to the curves of extension or deceleration.

The velocities u_f of free surface when the shock wave is reflected from it are of particular interest (see Fig. 1). For weak waves the well-known "doubling rule" holds: $u_f = 2u_H$. For shock waves of different intensities and various γ_0 the deviations from the doubling rule, that is, the values $u_f - 2u_H$, are given in Table 2. The upper dashed line bounds the wave amplitudes for which the relative error $\delta = u_f/(2u_H) - 1$ never exceeds 1% when the doubling rule is used. The lower line shows amplitudes which result in an error of less than 3%. For $u_H = 1.8$ and for all γ_0 one has $\delta \approx 10\%$.

4. Shock-Wave Attenuation

The dimensionless variables introduced in [10] can be employed to reduce the equations governing the one-dimensional motion of the medium in the plane to the dimensionless form. In fact, the continuity equations and equation of motion in these variables are

$$\partial x/\partial \tau + \partial u/\partial h = 0, \quad \partial u/\partial \tau + \partial p/\partial h = 0, \tag{4.1}$$

where $h = H/L$; $\tau = C_0 t_*/L$; L is the characteristic dimension of the problem; t_* is the time; H is the Lagrange coordinate, equal to the Euler coordinate Z if the matter is under normal conditions. The assumption the entropy of the particle found downstream of the shock front is conserved yields another equation:

$$\partial p/\partial x = G^2(x, p)\partial x/\partial \tau, \tag{4.2}$$

where $G(x, p) = c(x, p)/(1 - x)$. Thus, to determine the unknowns x, p, and u we have a system of three equations – (4.1) and (4.2) – depending on a single parameter γ_0 [the sound velocity c(x, p) depends on this parameter]. If one has to find the Euler coordinate, then (4.1) and (4.2) must be joined to another equation – $\partial z/\partial \tau = u$ – where $z = \lambda Z/L - (\lambda - 1)h$ is a dimensionless Euler coordinate.

TABLE 2

u_H \ γ_0	0,8	1,0	1,2	1,4	1,6	1,8	2,0
0,2	0,0000	0,0003	0,0005	0,0007	0,0009	0,0011	0,0013
0,3	0,0004	0,0011	0,0018	0,0024	0,0029	0,0035	0,0040
0,4	0,0014	0,0029	0,0044	0,0057	0,0068	0,0079	0,0089
0,5	0,0034	0,0062	0,0088	0,0110	0,0130	0,0148	0,0163
0,6	0,0068	0,0114	0,0154	0,0189	0,0219	0,0244	0,0265
0,8	0,0195	0,0291	0,0370	0,0434	0,0486	0,0527	0,0557
1,0	0,0431	0,0595	0,0723	0,0819	0,0889	0,0938	0,0970
1,2	0,0821	0,1067	0,1241	0,1360	0,1453	0,1477	0,1496
1,4	0,1423	0,1749	0,1952	0,2067	0,2123	0,2138	0,2125
1,6	0,2318	0,2687	0,2868	0,2939	0,2942	0,2904	0,2839
1,8	0,3622	0,3913	0,3988	0,3957	0,3872	0,3756	0,3623

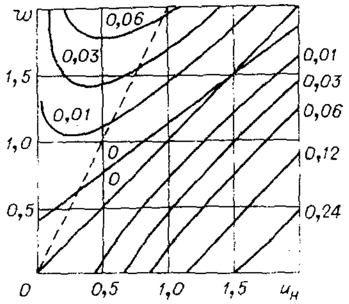

Fig. 4

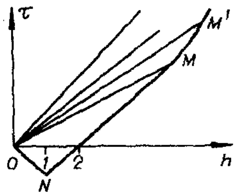

Fig. 5

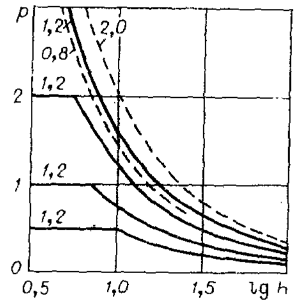

Fig. 6

The collision of a plate with a half-space is now analyzed; it is shown schematically in the $h-\tau$ plane in Fig. 5. In dimensionless coordinates, this problem depends only on the parameters γ_0 and the velocity $2u_0$ of the plate flying against it, where u_0 is the mass velocity at the shock front NM up to the instant $\tau = \tau_0$ at which the exhausted wave centered at the point O has caught up with the shock front and the attenuation of the shock wave starts. The point N corresponds to the instant of the impact of the plate and the half-space. The plate thickness in dimensionless coordinates is equal to 1; for τ_0 and the corresponding coordinate h_0 (the point M) one obtains

$$\tau_0 = 2(1 - x_0)/(c_H(x_0) - 1), \quad h_0 = \tau_0 G_H(x_0),$$

where $x_0 = x(h_0) = u_0/(1 + u_0)$; $c_H(x)$, $G_H(x)$ are the values of $c(x, p)$, $G_H(x, p)$ on the shock adiabat.

For the exhaustion wave one has the relations

$$dp/dx = G^2, \quad du/dx = G, \quad \xi \equiv G(x, \ p)\tau - h = 0.$$

The attenuations of the shock front, for example, the functions $p(h)$ and $x(h)$ along the length of the front, were evaluated by using the method of characteristics. A large number of calculations were carried out on an electronic computer for various values of γ_0 and $p_0 = x_0/(1 - x_0)^2$. Some of these results are shown in Fig. 6, where the values on the curves show the values of the parameter γ_0.

One now analyzes how the value of the quantity ξ changes along the length of the shock front for $\tau \geq \tau_0$. Of course, for $\tau = \tau_0$ one has $\xi = 0$. It is assumed that ξ when moving along the front length MM' varies sufficiently slowly for not very large values of p_0; ξ is expanded into a series in powers of $1/G_H - 1/G_H(x_0)$, and only the first (nonvanishing) term is retained. Then

$$G_H(x)\tau - h = \alpha[1/G_H(x) - 1/G_H(x_0)], \tag{4.3}$$

where α is a constant which depends on the parameters γ_0 and x_0. The constant α should be obtained from the original system of equations (4.1) and (4.2). By evaluating this constant and by adding to Eq. (4.3) the relation at the front, $d\tau/dh = 1 - x$, one obtains the following ordinary differential equation for the attenuation of the front:

$$d(h/h_0)/dx = F(x, \ h/h_0) \quad (h \geqslant h_0), \tag{4.4}$$

where

$$F(x, h/h_0) = \frac{q\,[2G_H(x_0)/G_H - 1] - h/h_0}{G_H[(1-x)G_H - 1]}\frac{dG_H}{dx};$$

$$G_H = G_H(x) = \sqrt{1 + x - \gamma_0 x^2}/(1 - x)^{3/2};$$

$$q = 1 - \frac{1}{2}\frac{(1 - x_0)G_H(x_0) + x_0 + 1}{1 + x_0 - \gamma_0 x_0^2}\frac{(2 + x_0)(2 - \gamma_0 x_0) - \gamma_0^2(1 - x_0)x_0^2}{(2 + x_0)(2 - \gamma_0 x_0)}.$$

If one ignores the changes of ξ when moving along the front and one sets $q = 0$, then Eq. (4.4) corresponds to the well-known isentropic approximation employed to solve the problem under consideration though with an equation of state which differs from the one in [13, 14]. If we take into account that the shock compression is not fully isentropic, we considerably improve the accuracy of the approximating formulas and widen the domain of their applications.

The front attenuation is better described by Eq. (4.4), the smaller p_0. But even for $p_0 = 3$, which in the case of copper corresponds to the pressure of $p_0 \approx 2.8$ Mbar, the comparison of the solution of (4.4) with the numerical computation by using the method of characteristics produces a relative error of $p(h)$ less than 2% in the region $0.8 \leq \gamma_0 \leq 2$ when the shock-front amplitude has been reduced 10 times.

For sufficiently small p_0 one obtains from (4.4) the approximate formula

$$x(h) = x_0 \Big/ \sqrt{1 + \frac{1 - \mu x_0^2}{1 + \mu x_0^2}\left(\frac{h}{h_0} - 1\right)}, \quad \mu = (3\gamma_0 + 1)/4,$$

which describes sufficiently well the function x(h) in the region $0.8 \le \gamma_0 \le 2$ for $p_0 \le 0.5$. Thus, if $p_0 = 0.5$ the formula yields a relative error for p(h) which is smaller than 5%.

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